Higher Witt K-theories

Jackson Morris (University of Washington)
joint w/ Kyle Ormsby (Reed College)

Cascade Topology Seminar
May 2026

Motivic Homotopy Theory

"Homotopy theory of schemes"

$$\mathsf{Sm}_F \longrightarrow \mathsf{Spc}_F = \mathsf{L}_{\mathbb{A}^1,\mathsf{Nis}}\mathsf{Fun}(\mathsf{Sm}_F^{\mathsf{op}}, \mathcal{S})$$

Presheaves of spaces satisfying

Nisnevich descent and $\mathbb{A}^1$- invariance

Motivic Homotopy Theory

"Homotopy theory of schemes"

$$\mathsf{Sm}_F \longrightarrow \mathsf{Spc}_F = \mathsf{L}_{\mathbb{A}^1,\mathsf{Nis}}\mathsf{Fun}(\mathsf{Sm}_F^{\mathsf{op}}, \mathcal{S})$$

Presheaves of spaces satisfying

Nisnevich descent and $\mathbb{A}^1$- invariance

It's not really that bad!

Yoneda embedding

$\mathsf{Sm}_F \hookrightarrow \mathsf{Spc}_F$

Constant functor

$\mathsf{Sm}_F \to \mathcal{S}$

Motivic Homotopy Theory

"Homotopy theory of schemes"

$$\mathrm{Sm}_F \longrightarrow \mathrm{Spc}_F = L_{\mathbb{A}^1,\mathrm{Nis}}\mathrm{Fun}(\mathrm{Sm}_F^{\mathrm{op}}, \mathcal{S})$$

Presheaves of spaces satisfying

Nisnevich descent and $\mathbb{A}^1$- invariance

It's not really that bad!

Yoneda embedding

$\mathrm{Sm}_F \hookrightarrow \mathrm{Spc}_F$

Constant functor

$\mathrm{Sm}_F \to \mathcal{S}$

$$\mathbb{P}^1 \simeq S^1 \wedge \mathbb{G}_m$$

Bigraded homotopy groups

$$S^{s,w} = (S^1)^{\wedge s-w} \wedge (\mathbb{G}_m)^{\wedge w}$$

$$\pi_{s,w}X = [S^{s,w}, X]_{\mathbb{A}^1}$$

Motivic Homotopy Theory

"Homotopy theory of schemes"

$\mathsf{Sm}_F \longrightarrow \mathsf{Spc}_F = \mathsf{L}_{\mathbb{A}^1,\mathsf{Nis}}\mathsf{Fun}(\mathsf{Sm}_F^{\mathsf{op}}, \mathcal{S})$

Presheaves of spaces satisfying

Nisnevich descent and $\mathbb{A}^1$- invariance

It's not really that bad!

Yoneda embedding

$\mathsf{Sm}_F \hookrightarrow \mathsf{Spc}_F$

Constant functor

$\mathsf{Sm}_F \to \mathcal{S}$

$\mathbb{P}^1 \simeq S^1 \wedge \mathbb{G}_m$

Bigraded homotopy groups

$S^{s,w} = (S^1)^{\wedge s-w} \wedge (\mathbb{G}_m)^{\wedge w}$

$\pi_{s,w}X = [S^{s,w}, X]_{\mathbb{A}^1}$

invert $\wedge \mathbb{P}^1$ $\longrightarrow$

Stable motivic homotopy theory

$\mathsf{SH}(F)$

Stable symmetric monoidal category

$\Sigma^\infty_{\mathbb{P}^1}\mathsf{Spec}(F) := \mathbb{S}$ motivic sphere spectrum

Motivic Homotopy Theory

"Homotopy theory of schemes"

$\mathsf{Sm}_F \longrightarrow \mathsf{Spc}_F = L_{\mathbb{A}^1,\mathsf{Nis}}\mathsf{Fun}(\mathsf{Sm}_F^{\mathsf{op}}, \mathcal{S})$

Presheaves of spaces satisfying

Nisnevich descent and $\mathbb{A}^1$- invariance

It's not really that bad!

Yoneda embedding

$\mathsf{Sm}_F \hookrightarrow \mathsf{Spc}_F$

Constant functor

$\mathsf{Sm}_F \to \mathcal{S}$

$\mathbb{P}^1 \simeq S^1 \wedge \mathbb{G}_m$

Bigraded homotopy groups

$S^{s,w} = (S^1)^{\wedge s-w} \wedge (\mathbb{G}_m)^{\wedge w}$

$\pi_{s,w}X = [S^{s,w}, X]_{\mathbb{A}^1}$

invert $\wedge \mathbb{P}^1$ $\longrightarrow$

Stable motivic homotopy theory

$\mathsf{SH}(F)$

Stable symmetric monoidal category

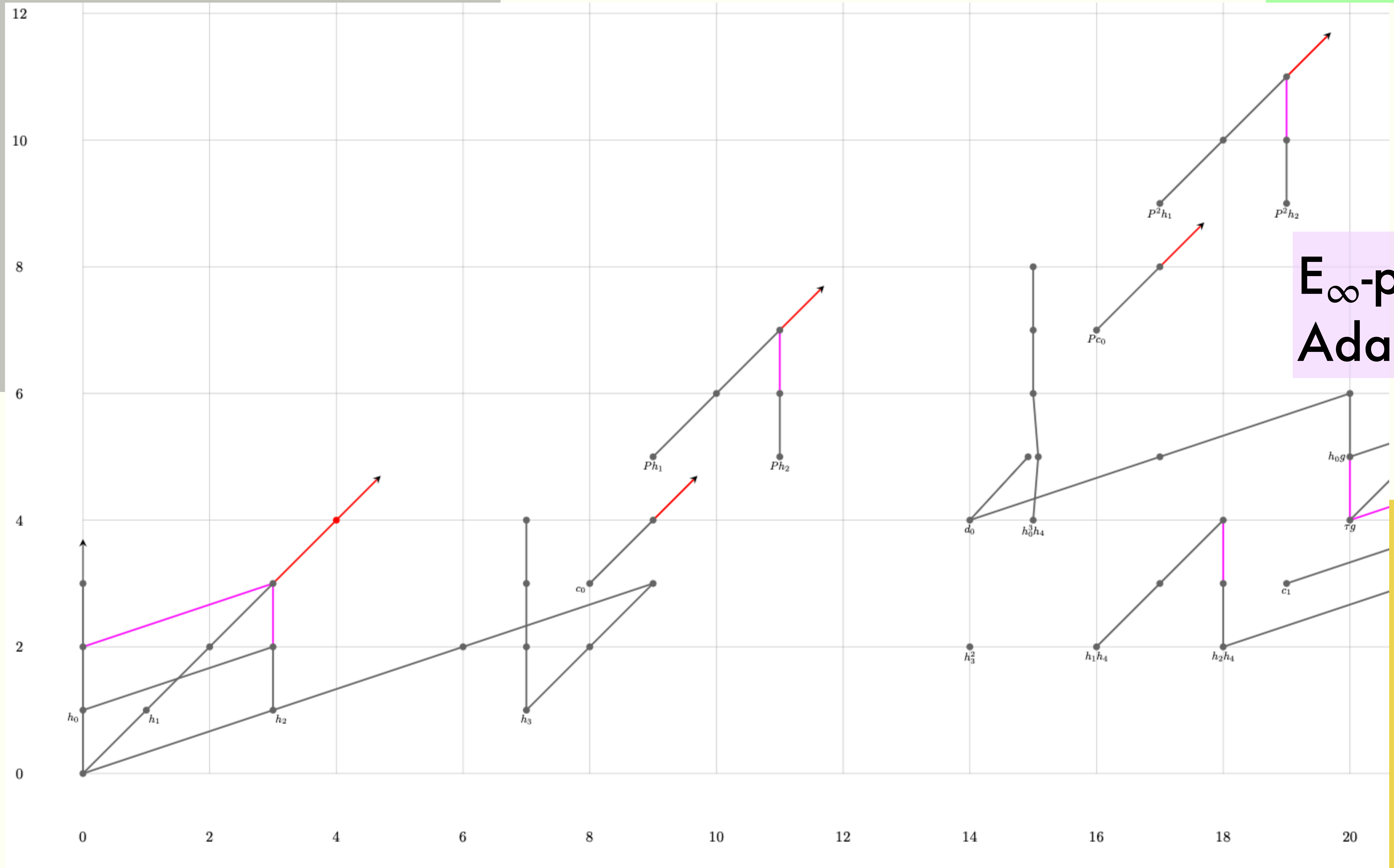
$\Sigma^\infty_{\mathbb{P}^1}\mathsf{Spec}(F) := \mathbb{S}$ motivic sphere spectrum

Goal: understand $\pi^F_{**}(\mathbb{S})$

Goal: understand $\pi_{**}^{F}(\mathbb{S})$

Goal: understand $\pi_{**}^{\mathbb{C}}(\mathbb{S})$

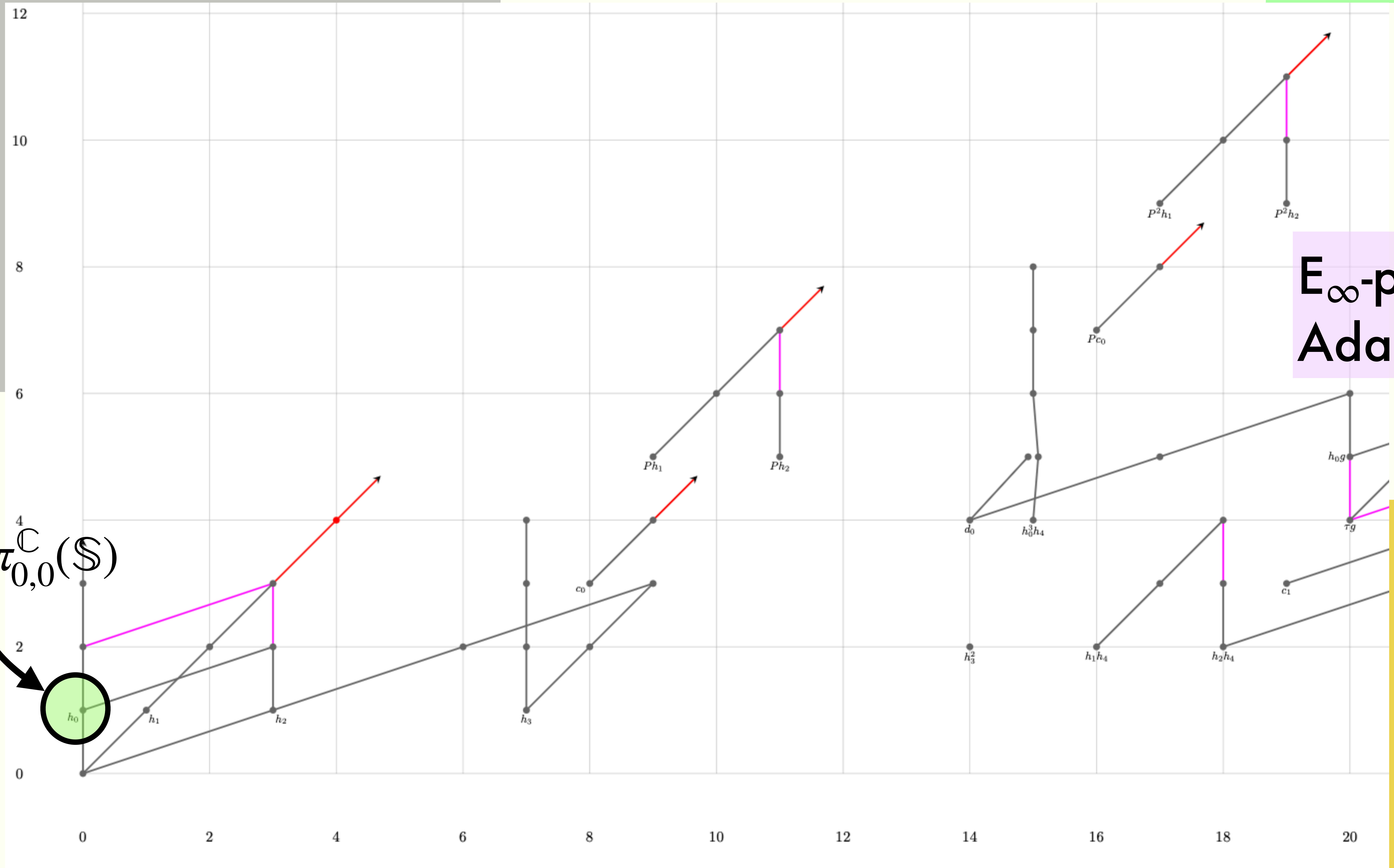
Goal: understand $\pi^{\mathbb{C}}_{**}(\mathbb{S})$



E_∞-page of the motivic Adams spectral sequence

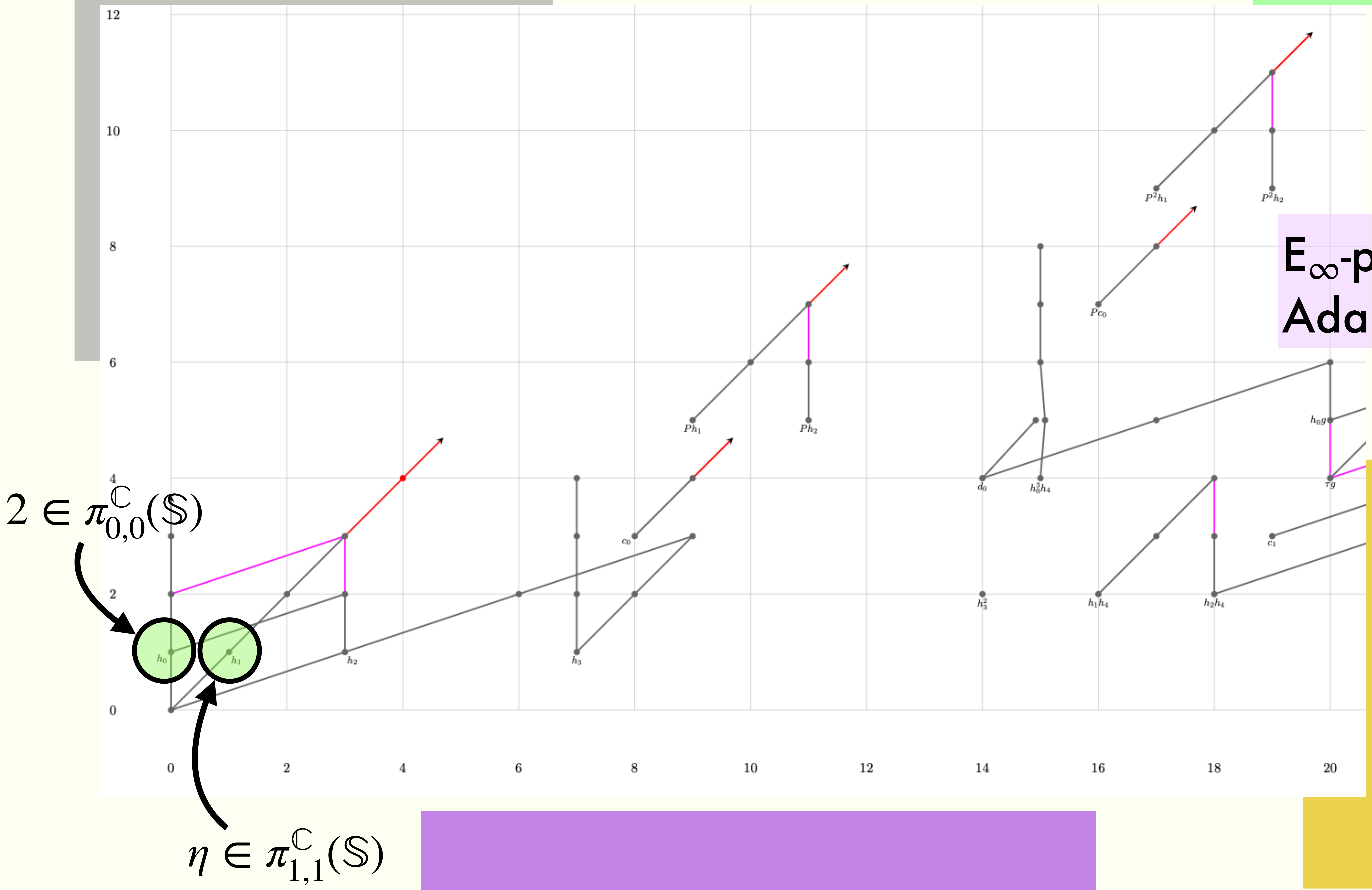
Goal: understand $\pi^{\mathbb{C}}_{**}(\mathbb{S})$

$2 \in \pi^{\mathbb{C}}_{0,0}(\mathbb{S})$



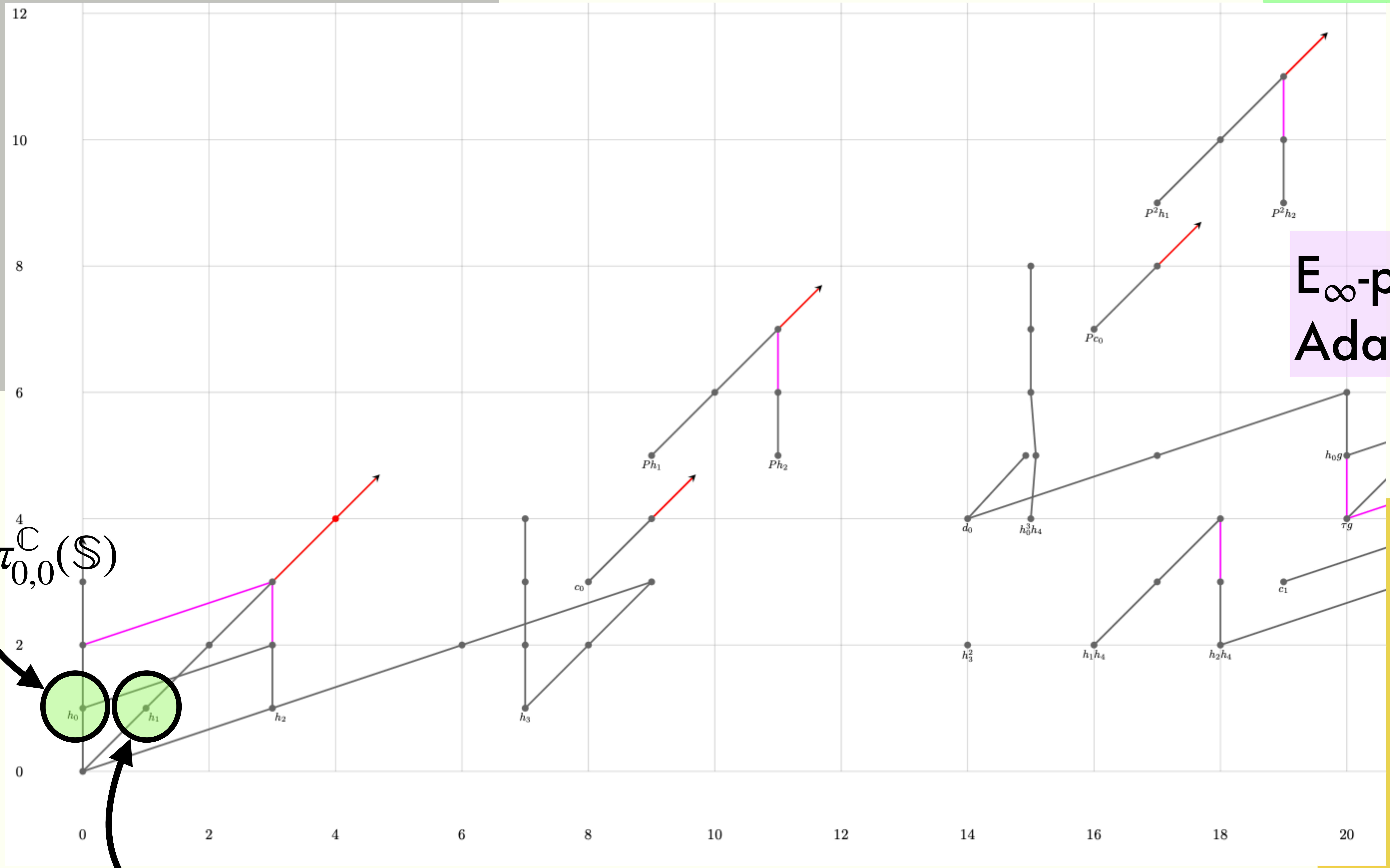
E_∞-page of the motivic Adams spectral sequence

Goal: understand $\pi_{**}^{\mathbb{C}}(\mathbb{S})$



E_∞-page of the motivic Adams spectral sequence

Goal: understand $\pi^{\mathbb{C}}_{**}(\mathbb{S})$



E_∞-page of the motivic Adams spectral sequence

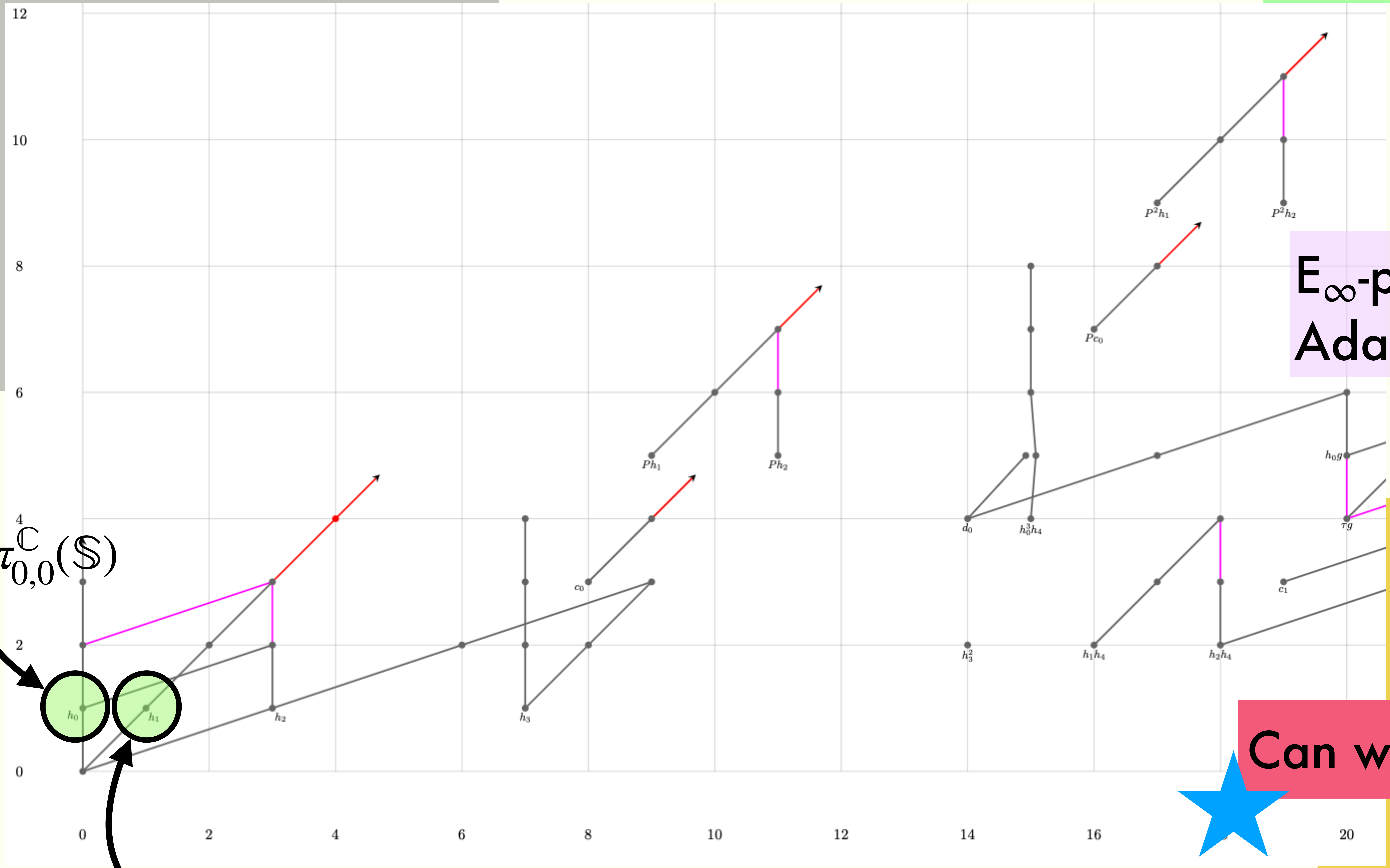
$2 \in \pi^{\mathbb{C}}_{0,0}(\mathbb{S})$

$\eta \in \pi^{\mathbb{C}}_{1,1}(\mathbb{S})$

This is very hard/confusing...

Goal: understand $\pi^{\mathbb{C}}_{**}(\mathbb{S})$

E_∞-page of the motivic Adams spectral sequence



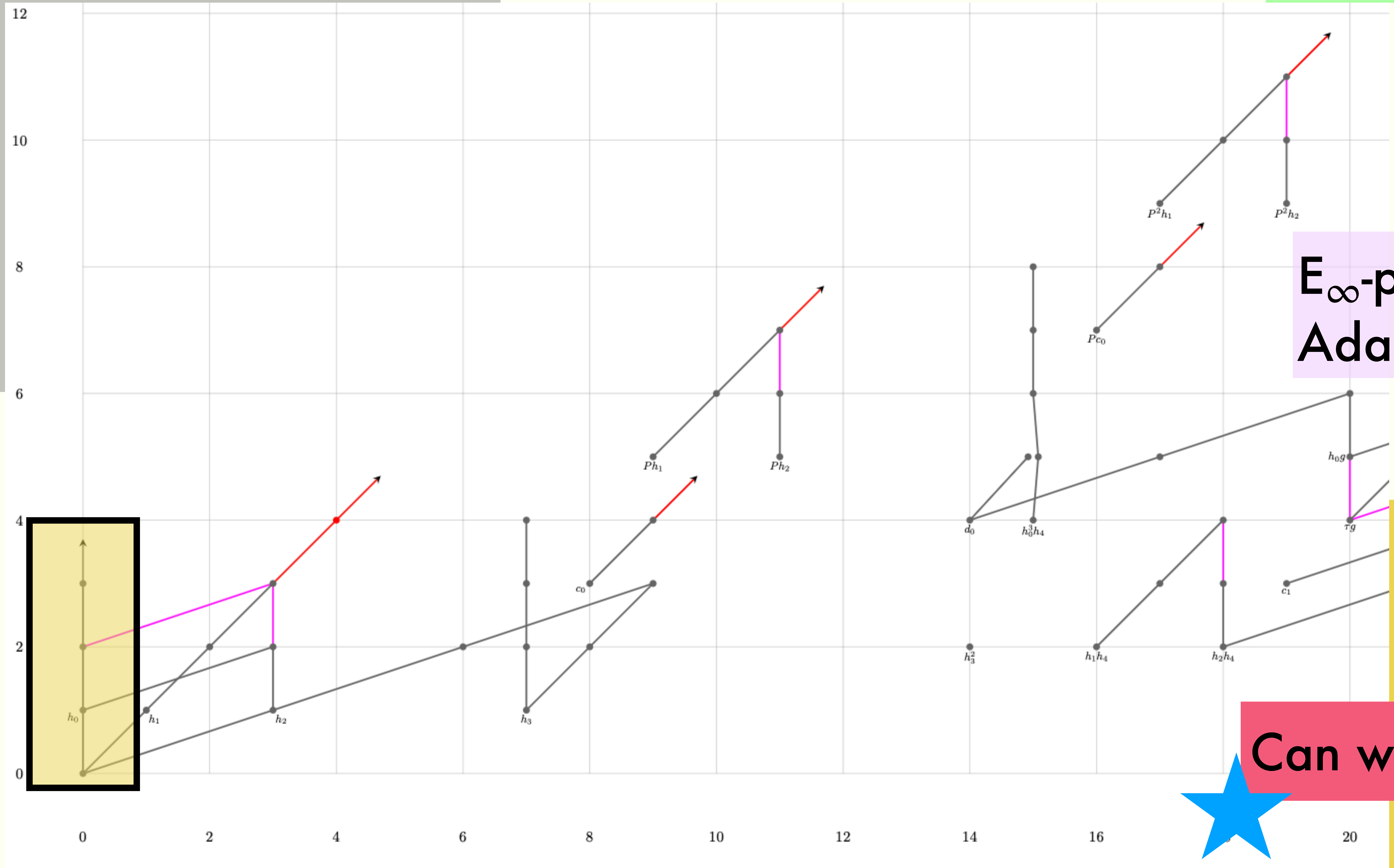
$2 \in \pi^{\mathbb{C}}_{0,0}(\mathbb{S})$

$\eta \in \pi^{\mathbb{C}}_{1,1}(\mathbb{S})$

This is very hard/confusing...

Can we find any patterns?

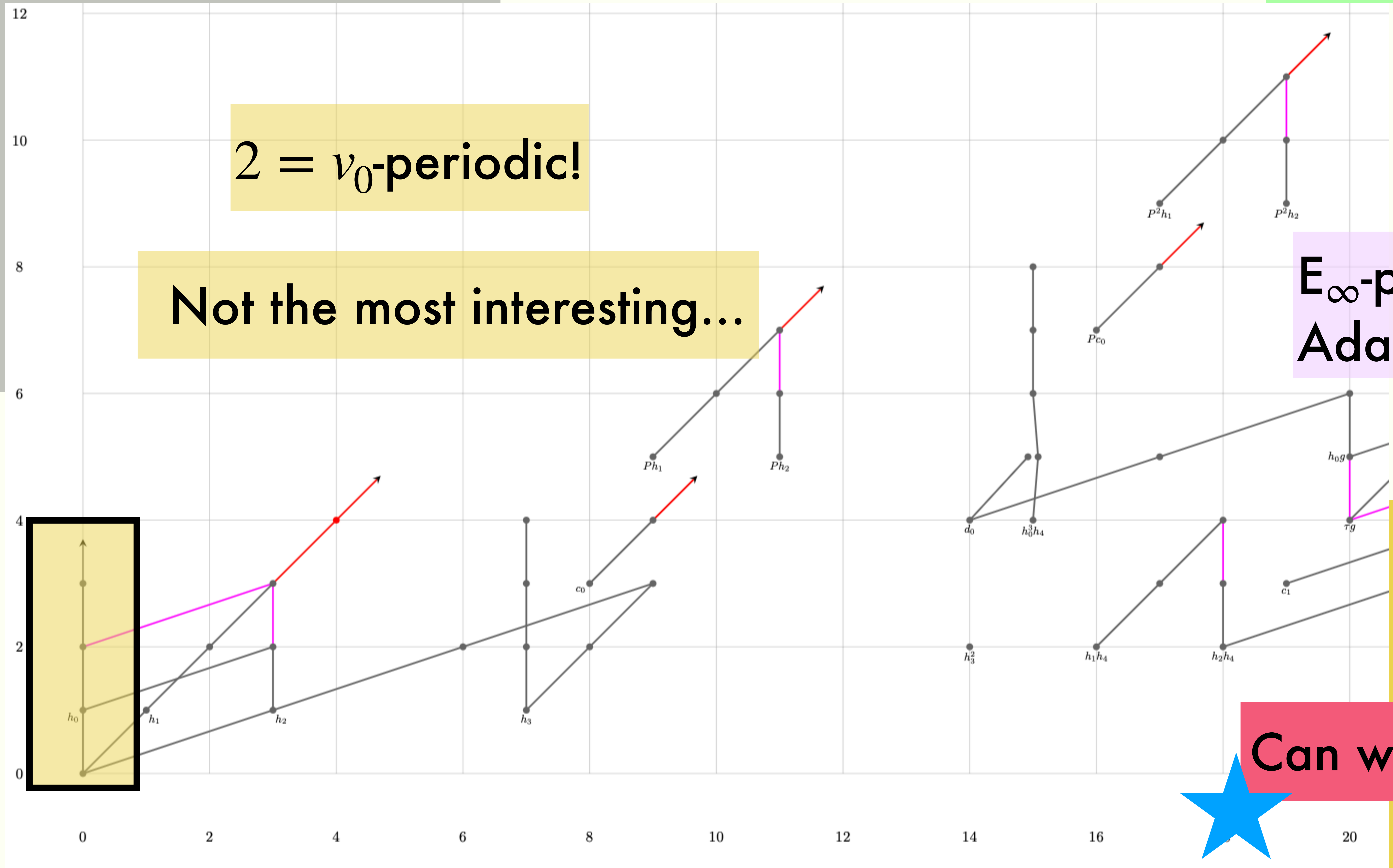
Goal: understand $\pi_{**}^{\mathbb{C}}(\mathbb{S})$



E_∞-page of the motivic Adams spectral sequence

Can we find any patterns?

Goal: understand $\pi_{**}^{\mathbb{C}}(\mathbb{S})$



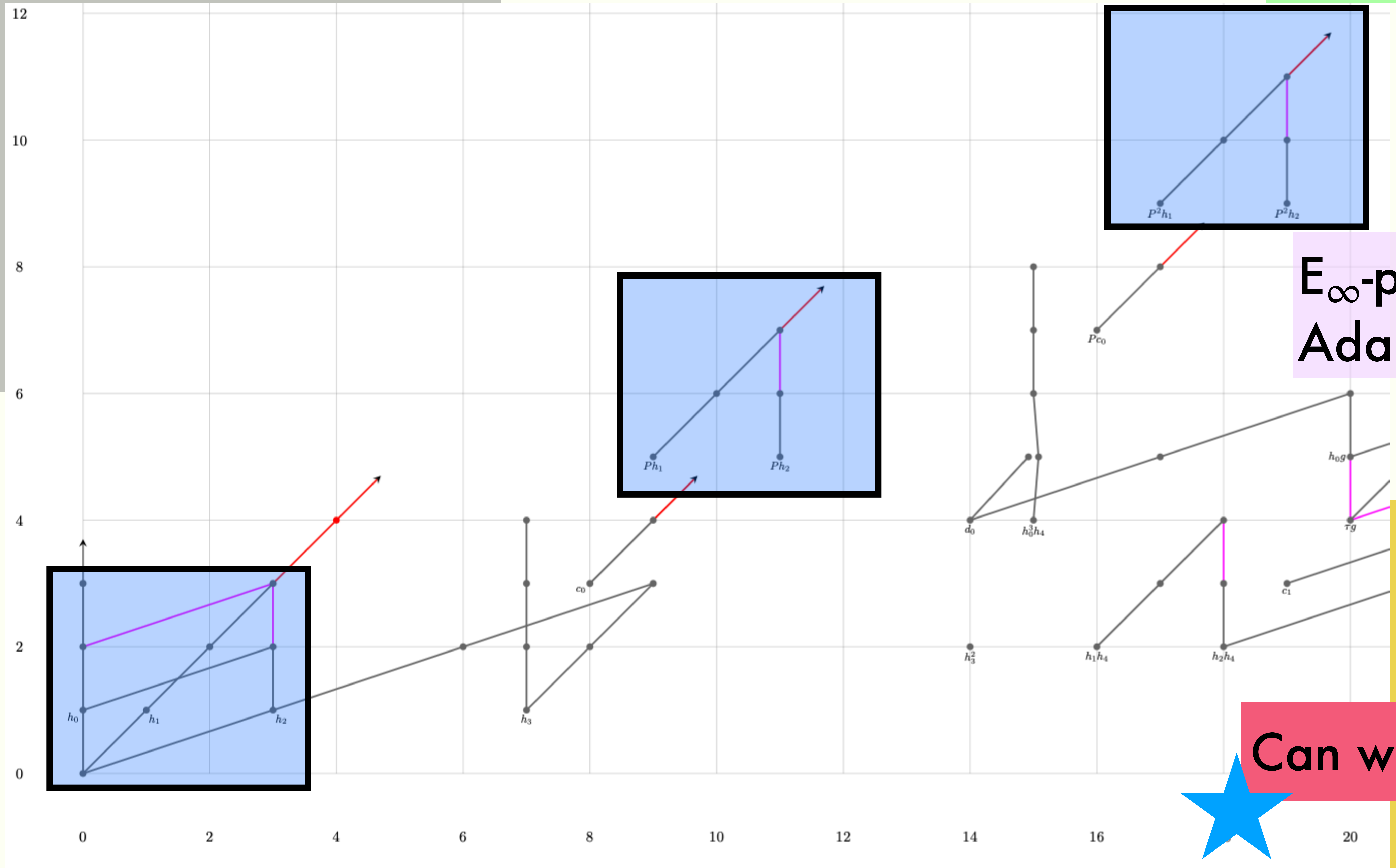
$2 = v_0$-periodic!

Not the most interesting...

E_∞-page of the motivic Adams spectral sequence

Can we find any patterns?

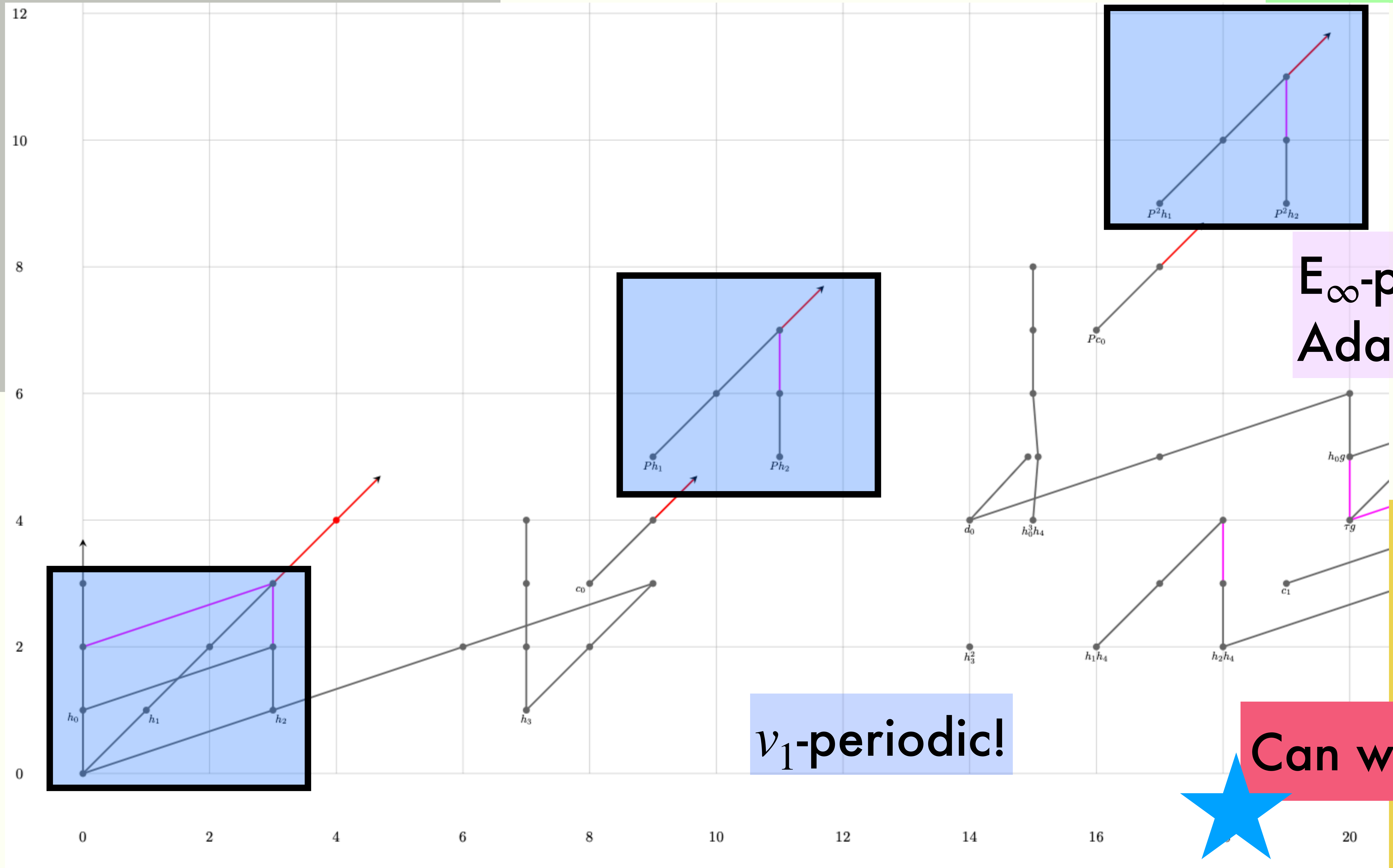
Goal: understand $\pi_{**}^{\mathbb{C}}(\mathbb{S})$



E_∞-page of the motivic Adams spectral sequence

Can we find any patterns?

Goal: understand $\pi^{\mathbb{C}}_{**}(\mathbb{S})$

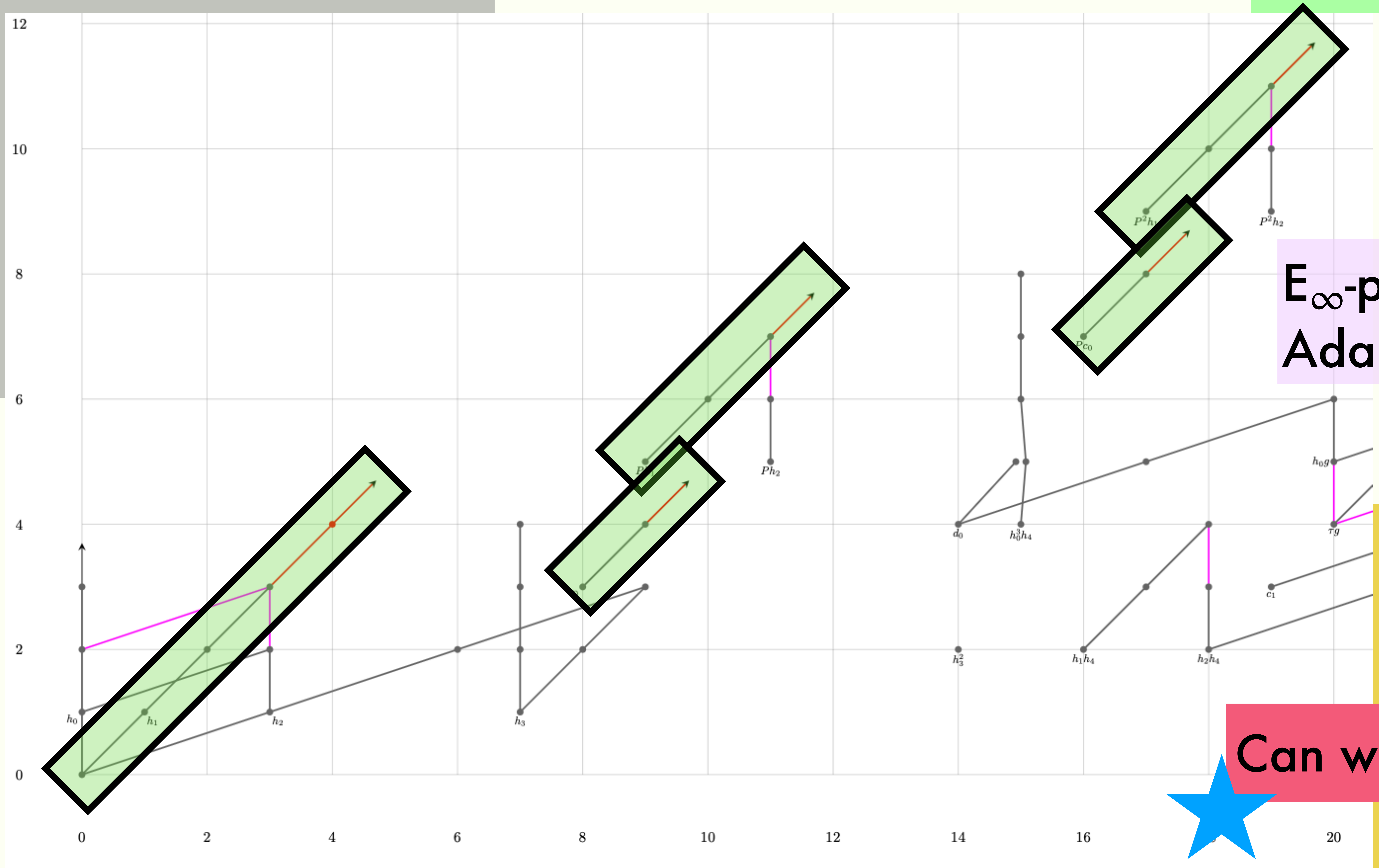


E_∞-page of the motivic Adams spectral sequence

v_1-periodic!

Can we find any patterns?

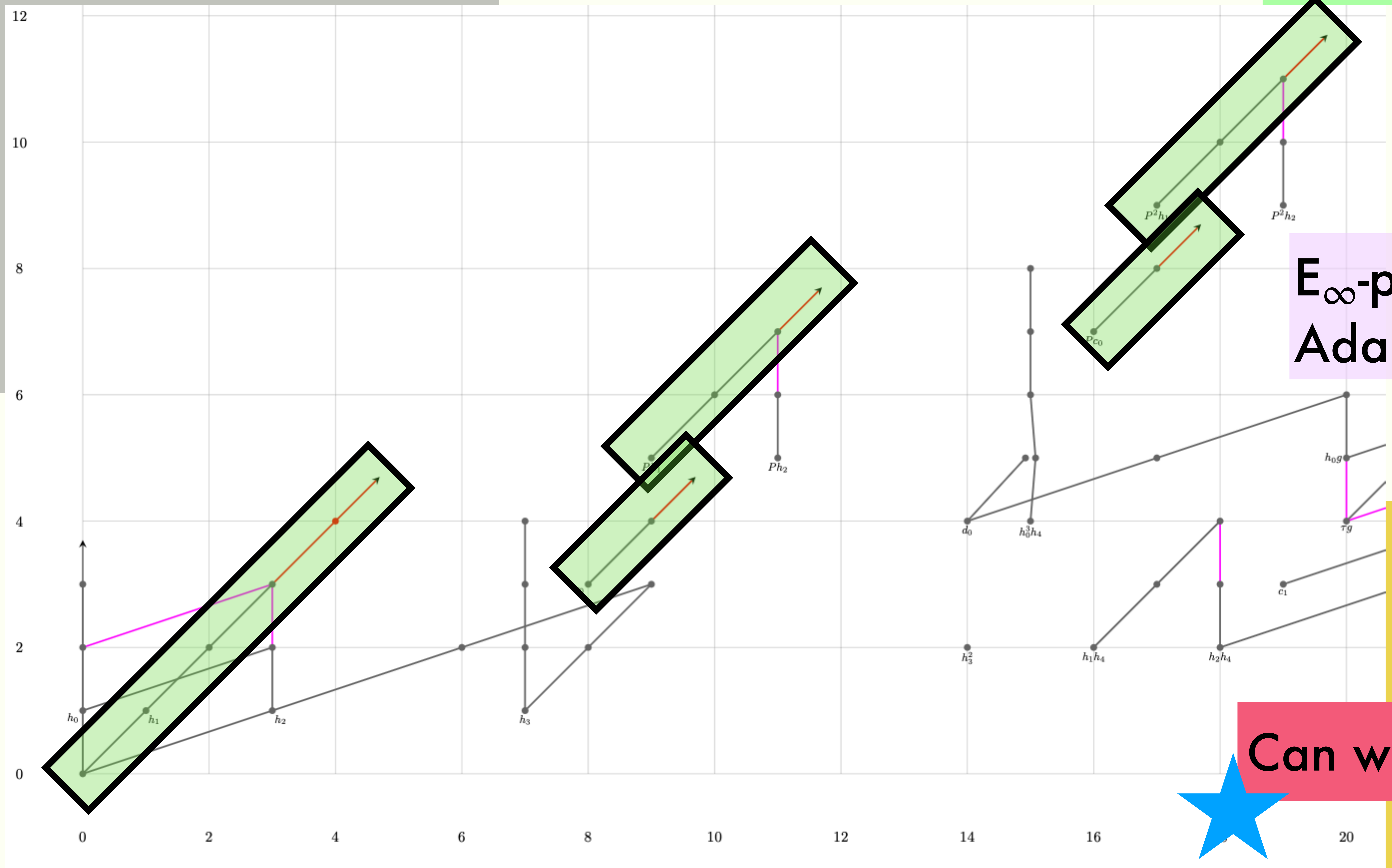
Goal: understand $\pi_{**}^{\mathbb{C}}(\mathbb{S})$



E_∞-page of the motivic Adams spectral sequence

Can we find any patterns?

Goal: understand $\pi_{**}^{\mathbb{C}}(\mathbb{S})$



E_∞-page of the motivic Adams spectral sequence

Can we find any patterns?

η-periodic!

Question: How do we compute these periodic layers of $\pi_{**}^{\mathbb{C}}(\mathbb{S})$?

Question: How do we compute these periodic layers of $\pi_{**}^{\mathbb{C}}(\mathbb{S})$?

Take a leaf out of the chromatic notebook

Question: How do we compute these periodic layers of $\pi_{**}^{\mathbb{C}}(\mathbb{S})$?

Take a leaf out of the chromatic notebook

Consider the motivic Lubin-Tate spectrum E_n

Question: How do we compute these periodic layers of $\pi^{\mathbb{C}}_{**}(\mathbb{S})$?

Take a leaf out of the chromatic notebook

Consider the motivic Lubin-Tate spectrum E_n

$$\pi^{\mathbb{C}}_{**}(\mathsf{E}_n) = \mathsf{W}(\mathbb{F}_{2^n})[[u_1, \ldots, u_{n-1}]][u^{\pm 1}, \tau]$$

Question: How do we compute these periodic layers of $\pi_{**}^{\mathbb{C}}(\mathbb{S})$?

Take a leaf out of the chromatic notebook

Consider the motivic Lubin-Tate spectrum E_n

$$\pi_{**}^{\mathbb{C}}(\mathsf{E}_n) = \mathsf{W}(\mathbb{F}_{2^n})[[u_1, \ldots, u_{n-1}]][u^{\pm 1}, \tau]$$

$\mathbb{S} \to \mathsf{E}_n$ only sees v_0-periodic data

Question: How do we compute these periodic layers of $\pi^{\mathbb{C}}_{**}(\mathbb{S})$?

Take a leaf out of the chromatic notebook

Consider the motivic Lubin-Tate spectrum E_n

$$\pi^{\mathbb{C}}_{**}(\mathsf{E}_n) = \mathsf{W}(\mathbb{F}_{2^n})[[u_1, \ldots, u_{n-1}]][u^{\pm 1}, \tau]$$

$\mathbb{S} \to \mathsf{E}_n$ only sees v_0-periodic data

C_2-action

Question: How do we compute these periodic layers of $\pi_{**}^{\mathbb{C}}(\mathbb{S})$?

Take a leaf out of the chromatic notebook

Consider the motivic Lubin-Tate spectrum E_n

$$\pi_{**}^{\mathbb{C}}(\mathsf{E}_n) = \mathsf{W}(\mathbb{F}_{2^n})[[u_1, \ldots, u_{n-1}]][u^{\pm 1}, \tau]$$

$\mathbb{S} \to \mathsf{E}_n$ only sees v_0-periodic data

C_2-action

The fixed points $\mathsf{E}_n^{hC_2}$ see more!

Question: How do we compute these periodic layers of $\pi_{**}^{\mathbb{C}}(\mathbb{S})$?

Take a leaf out of the chromatic notebook

Consider the motivic Lubin-Tate spectrum E_n

$$\pi_{**}^{\mathbb{C}}(\mathsf{E}_n) = \mathsf{W}(\mathbb{F}_{2^n})[[u_1, \ldots, u_{n-1}]][u^{\pm 1}, \tau]$$

$\mathbb{S} \to \mathsf{E}_n$ only sees v_0-periodic data

C_2-action

The fixed points $\mathsf{E}_n^{hC_2}$ see more!

For n=1, we have that

$\mathsf{E}_1 = KGL$, algebraic K-theory spectrum

Question: How do we compute these periodic layers of $\pi_{**}^{\mathbb{C}}(\mathbb{S})$?

Take a leaf out of the chromatic notebook

Consider the motivic Lubin-Tate spectrum E_n

$$\pi_{**}^{\mathbb{C}}(\mathsf{E}_n) = \mathsf{W}(\mathbb{F}_{2^n})[[u_1, \ldots, u_{n-1}]][u^{\pm 1}, \tau]$$

$\mathbb{S} \to \mathsf{E}_n$ only sees v_0-periodic data

C_2-action

The fixed points $\mathsf{E}_n^{hC_2}$ see more!

For n=1, we have that

$\mathsf{E}_1 = KGL$, algebraic K-theory spectrum

<u>Thm</u> [Hu-Kriz-Ormsby]

$\mathsf{E}_1^{hC_2} = KQ$, the hermitian K-theory spectrum

Question: How do we compute these periodic layers of $\pi_{**}^{\mathbb{C}}(\mathbb{S})$?

Take a leaf out of the chromatic notebook

Consider the motivic Lubin-Tate spectrum E_n

$$\pi_{**}^{\mathbb{C}}(\mathsf{E}_n) = \mathsf{W}(\mathbb{F}_{2^n})[[u_1, \ldots, u_{n-1}]][u^{\pm 1}, \tau]$$

$\mathbb{S} \to \mathsf{E}_n$ only sees v_0-periodic data

C_2-action

The fixed points $\mathsf{E}_n^{hC_2}$ see more!

For n=1, we have that

$\mathsf{E}_1 = KGL$, algebraic K-theory spectrum

Thm [Hu-Kriz-Ormsby]

$\mathsf{E}_1^{hC_2} = KQ$, the hermitian K-theory spectrum

Thm [Balderrama-Ormsby-Quigley]

There is a cofiber sequence

$\mathbb{S}_{KGL/2} \to KQ \to KQ$

Question: How do we compute these periodic layers of $\pi^{\mathbb{C}}_{**}(\mathbb{S})$?

Consider the motivic Lubin-Tate spectrum E_n

$$\pi^{\mathbb{C}}_{**}(\mathsf{E}_n) = \mathsf{W}(\mathbb{F}_{2^n})[[u_1, \ldots, u_{n-1}]][u^{\pm 1}, \tau]$$

$\mathbb{S} \to \mathsf{E}_n$ only sees v_0-periodic data

C_2-action

The fixed points $\mathsf{E}_n^{hC_2}$ see more!

For n=1, we have that

$\mathsf{E}_1 = KGL$, algebraic K-theory spectrum

<u>Thm</u> [Hu-Kriz-Ormsby]

$\mathsf{E}_1^{hC_2} = KQ$, the hermitian K-theory spectrum

Take a leaf out of the chromatic notebook

<u>Thm</u> [Balderrama-Ormsby-Quigley]

There is a cofiber sequence

$\mathbb{S}_{KGL/2} \to KQ \to KQ$

$\pi^{\mathbb{C}}_{**}(\mathbb{S}_{KGL/2})$ captures the v_1-periodicity of the sphere!

Question: How do we compute these periodic layers of $\pi^{\mathbb{C}}_{**}(\mathbb{S})$?

Take a leaf out of the chromatic notebook

Consider the motivic Lubin-Tate spectrum E_n

$$\pi^{\mathbb{C}}_{**}(\mathsf{E}_n) = \mathsf{W}(\mathbb{F}_{2^n})[[u_1, \ldots, u_{n-1}]][u^{\pm 1}, \tau]$$

$\mathbb{S} \to \mathsf{E}_n$ only sees v_0-periodic data

C_2-action

The fixed points $\mathsf{E}_n^{hC_2}$ see more!

For n=1, we have that

$\mathsf{E}_1 = KGL$, algebraic K-theory spectrum

Thm [Hu-Kriz-Ormsby]

$\mathsf{E}_1^{hC_2} = KQ$, the hermitian K-theory spectrum

Thm [Balderrama-Ormsby-Quigley]

There is a cofiber sequence

$\mathbb{S}_{KGL/2} \to KQ \to KQ$

$\pi^{\mathbb{C}}_{**}(\mathbb{S}_{KGL/2})$ captures the v_1-periodicity of the sphere!

What about η-periodicity?

What about η-periodicity?

What about η-periodicity?

Thm (Andrews-Miller)

$$\pi_{**}^{\mathbb{C}}(\eta^{-1}\mathbb{S}) \cong \mathbb{F}_2[\eta^{\pm 1}, \sigma, \mu_9]/(\eta\sigma^2)$$

What about η-periodicity?

<u>Thm (Andrews-Miller)</u>

$\pi^{\mathbb{C}}_{**}(\eta^{-1}\mathbb{S}) \cong \mathbb{F}_2[\eta^{\pm 1}, \sigma, \mu_9]/(\eta\sigma^2)$

<u>Thm (Bachmann-Hopkins)</u>

There is a cofiber sequence

$\eta^{-1}\mathbb{S} \to kw \to \Sigma^4 kw$, where

$$kw = \tau_{\geq 0}(\eta^{-1}KQ)$$

What about η-periodicity?

Just inverting an element…
kinda like v_0-periodicity

Thm (Andrews-Miller)
$\pi^{\mathbb{C}}_{**}(\eta^{-1}\mathbb{S}) \cong \mathbb{F}_2[\eta^{\pm 1}, \sigma, \mu_9]/(\eta\sigma^2)$

Thm (Bachmann-Hopkins)
There is a cofiber sequence
$\eta^{-1}\mathbb{S} \to kw \to \Sigma^4 kw$, where

$$kw = \tau_{\geq 0}(\eta^{-1}KQ)$$

What about η-periodicity?

Just inverting an element...
kinda like v_0-periodicity

Thm (Andrews-Miller)
$\pi^{\mathbb{C}}_{**}(\eta^{-1}\mathbb{S}) \cong \mathbb{F}_2[\eta^{\pm 1}, \sigma, \mu_9]/(\eta\sigma^2)$

Thm (Bachmann-Hopkins)
There is a cofiber sequence
$\eta^{-1}\mathbb{S} \to kw \to \Sigma^4 kw$, where

$$kw = \tau_{\geq 0}(\eta^{-1}KQ)$$

Kinda like inverting η on
$\mathbb{S}_{KGL/2} \to KQ \to KQ \ldots$

What about η-periodicity?

Just inverting an element...
kinda like v_0-periodicity

Thm (Andrews-Miller)
$\pi^{\mathbb{C}}_{**}(\eta^{-1}\mathbb{S}) \cong \mathbb{F}_2[\eta^{\pm 1}, \sigma, \mu_9]/(\eta\sigma^2)$

Question: What the heck?

Thm (Bachmann-Hopkins)
There is a cofiber sequence
$\eta^{-1}\mathbb{S} \to kw \to \Sigma^4 kw$, where

$$kw = \tau_{\geq 0}(\eta^{-1}KQ)$$

Kinda like inverting η on
$\mathbb{S}_{KGL/2} \to KQ \to KQ$...

What about η-periodicity?

Just inverting an element...
kinda like v_0-periodicity

Thm (Andrews-Miller)
$\pi^{\mathbb{C}}_{**}(\eta^{-1}\mathbb{S}) \cong \mathbb{F}_2[\eta^{\pm 1}, \sigma, \mu_9]/(\eta\sigma^2)$

Question: What the heck?

Answer: exotic periodicity!

Thm (Bachmann-Hopkins)
There is a cofiber sequence
$\eta^{-1}\mathbb{S} \to kw \to \Sigma^4 kw$, where

$$kw = \tau_{\geq 0}(\eta^{-1}KQ)$$

Kinda like inverting η on
$\mathbb{S}_{KGL/2} \to KQ \to KQ\ldots$

Answer: exotic periodicity!

Answer: exotic periodicity!

Should think of $\eta = w_0$ in a new family of w_n-periodicity

Answer: exotic periodicity!

Should think of $\eta = w_0$ in a new family of w_n-periodicity

Thm (Andrews)
There is a non-nilpotent self-map $w_1^4 : \Sigma^{20,12}\mathbb{S}/\eta \to \mathbb{S}/\eta$

Answer: exotic periodicity!

Should think of $\eta = w_0$ in a new family of w_n-periodicity

Thm (Andrews)
There is a non-nilpotent self-map
$w_1^4 : \Sigma^{20,12}\mathbb{S}/\eta \to \mathbb{S}/\eta$

w_1-periodicity in the sphere!

Answer: exotic periodicity!

Should think of $\eta = w_0$ in a new family of w_n-periodicity

Thm (Andrews)
There is a non-nilpotent self-map $w_1^4 : \Sigma^{20,12}\mathbb{S}/\eta \to \mathbb{S}/\eta$

w_1-periodicity in the sphere!

Very complicated.

Answer: exotic periodicity!

Should think of $\eta = w_0$ in a new family of w_n-periodicity

<u>Thm (Andrews)</u>
There is a non-nilpotent self-map
$w_1^4 : \Sigma^{20,12}\mathbb{S}/\eta \to \mathbb{S}/\eta$

w_1-periodicity in the sphere!

Very complicated.

We saw how the cofiber sequence for v_1-periodicity can be deformed to a cofiber sequence for w_0-periodicity

Answer: exotic periodicity!

Should think of $\eta = w_0$ in a new family of w_n-periodicity

Thm (Andrews)
There is a non-nilpotent self-map
$w_1^4 : \Sigma^{20,12}\mathbb{S}/\eta \to \mathbb{S}/\eta$

w_1-periodicity in the sphere!

Very complicated.

We saw how the cofiber sequence for v_1-periodicity can be deformed to a cofiber sequence for w_0-periodicity

$$\mathbb{S}_{KGL/2} \to KQ \to KQ$$

$$\eta^{-1}\mathbb{S} \to kw \to \Sigma^4 kw$$

Answer: exotic periodicity!

Should think of $\eta = w_0$ in a new family of w_n-periodicity

Thm (Andrews)
There is a non-nilpotent self-map
$w_1^4 : \Sigma^{20,12}\mathbb{S}/\eta \to \mathbb{S}/\eta$

w_1-periodicity in the sphere!

Very complicated.

We saw how the cofiber sequence for v_1-periodicity can be deformed to a cofiber sequence for w_0-periodicity

$$\mathbb{S}_{KGL/2} \to KQ \to KQ$$

Thm [Hu-Kriz-Ormsby]
$\mathrm{E}_1^{hC_2} = KQ$, the hermitian K-theory spectrum

$$\eta^{-1}\mathbb{S} \to kw \to \Sigma^4 kw$$

Answer: exotic periodicity!

Should think of $\eta = w_0$ in a new family of w_n-periodicity

Thm (Andrews)
There is a non-nilpotent self-map
$w_1^4 : \Sigma^{20,12}\mathbb{S}/\eta \to \mathbb{S}/\eta$

w_1-periodicity in the sphere!

Very complicated.

We saw how the cofiber sequence for v_1-periodicity can be deformed to a cofiber sequence for w_0-periodicity

$$\mathbb{S}_{KGL/2} \to KQ \to KQ$$

Thm [Hu-Kriz-Ormsby]
$\mathrm{E}_1^{hC_2} = KQ$, the hermitian K-theory spectrum
$\mathrm{E}_1^{tC_2} = KW = \eta^{-1}KQ$, the Witt K-theory spectrum

$$\eta^{-1}\mathbb{S} \to kw \to \Sigma^4 kw$$

Answer: exotic periodicity!

Should think of $\eta = w_0$ in a new family of w_n-periodicity

Thm (Andrews)
There is a non-nilpotent self-map
$w_1^4 : \Sigma^{20,12}\mathbb{S}/\eta \to \mathbb{S}/\eta$

w_1-periodicity in the sphere!

Very complicated.

We saw how the cofiber sequence for v_1-periodicity can be deformed to a cofiber sequence for w_0-periodicity

$$\mathbb{S}_{KGL/2} \to KQ \to KQ$$

Thm [Hu-Kriz-Ormsby]
$\mathrm{E}_1^{hC_2} = KQ$, the hermitian K-theory spectrum
$\mathrm{E}_1^{tC_2} = KW = \eta^{-1}KQ$, the Witt K-theory spectrum

$$\eta^{-1}\mathbb{S} \to kw \to \Sigma^4 kw$$

Idea: study Tate fixed points of E_n

Tate fixed points decrease chromatic height and shift to a different periodic family

Tate fixed points decrease chromatic height and shift to a different periodic family

Call this Diachromatic Blueshift

Tate fixed points decrease chromatic height and shift to a different periodic family

Call this Diachromatic Blueshift

Let $\mathrm{EW}_{n-1} = (\mathrm{E}_n)^{tC_2}$ denote higher Witt K-theory.

Let $\mathrm{EQ}_{n-1} = (E_n)^{hC_2}$ denote higher hermitian K-theory.

Tate fixed points decrease chromatic height and shift to a different periodic family

Call this Diachromatic Blueshift

Thm (M-Ormsby):

1. The spectrum EW_{n-1} is nontrivial.

Let $EW_{n-1} = (E_n)^{tC_2}$ denote higher Witt K-theory.

Let $EQ_{n-1} = (E_n)^{hC_2}$ denote higher hermitian K-theory.

Tate fixed points decrease chromatic height and shift to a different periodic family

Call this Diachromatic Blueshift

Thm (M-Ormsby):

1. The spectrum EW_{n-1} is nontrivial.

2. Some power of each of $\eta = w_0, \ldots, w_{n-1}$ act by isomorphisms.

Let $\mathrm{EW}_{n-1} = (\mathrm{E}_n)^{tC_2}$ denote higher Witt K-theory.

Let $\mathrm{EQ}_{n-1} = (E_n)^{hC_2}$ denote higher hermitian K-theory.

Tate fixed points decrease chromatic height and shift to a different periodic family

Call this Diachromatic Blueshift

Thm (M-Ormsby):

1. The spectrum EW_{n-1} is nontrivial.

2. Some power of each of $\eta = w_0, \ldots, w_{n-1}$ act by isomorphisms.

3. $\eta^{-1}\mathrm{EQ}_{n-1} = w_1^{-1}\mathrm{EQ}_{n-1} = \ldots = w_{n-1}^{-1}\mathrm{EQ}_{n-1} = \mathrm{EW}_{n-1}$.

Let $\mathrm{EW}_{n-1} = (\mathrm{E}_n)^{tC_2}$ denote higher Witt K-theory.

Let $\mathrm{EQ}_{n-1} = (E_n)^{hC_2}$ denote higher hermitian K-theory.

Tate fixed points decrease chromatic height and shift to a different periodic family

Call this Diachromatic Blueshift

Thm (M-Ormsby):

1. The spectrum EW_{n-1} is nontrivial.

2. Some power of each of $\eta = w_0, \ldots, w_{n-1}$ act by isomorphisms.

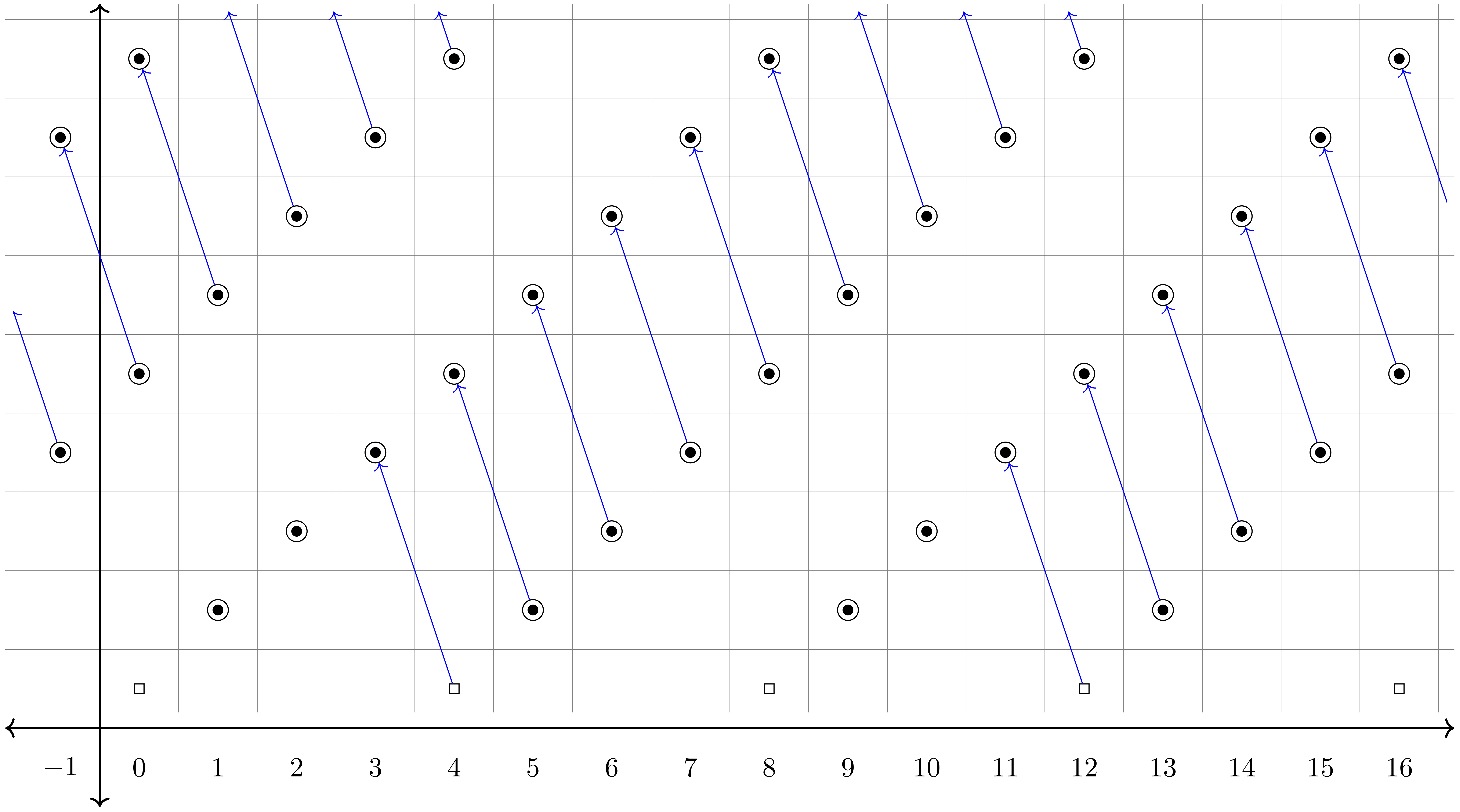
3. $\eta^{-1}\mathrm{EQ}_{n-1} = w_1^{-1}\mathrm{EQ}_{n-1} = \ldots = w_{n-1}^{-1}\mathrm{EQ}_{n-1} = \mathrm{EW}_{n-1}$.

Let $\mathrm{EW}_{n-1} = (\mathrm{E}_n)^{tC_2}$ denote higher Witt K-theory.

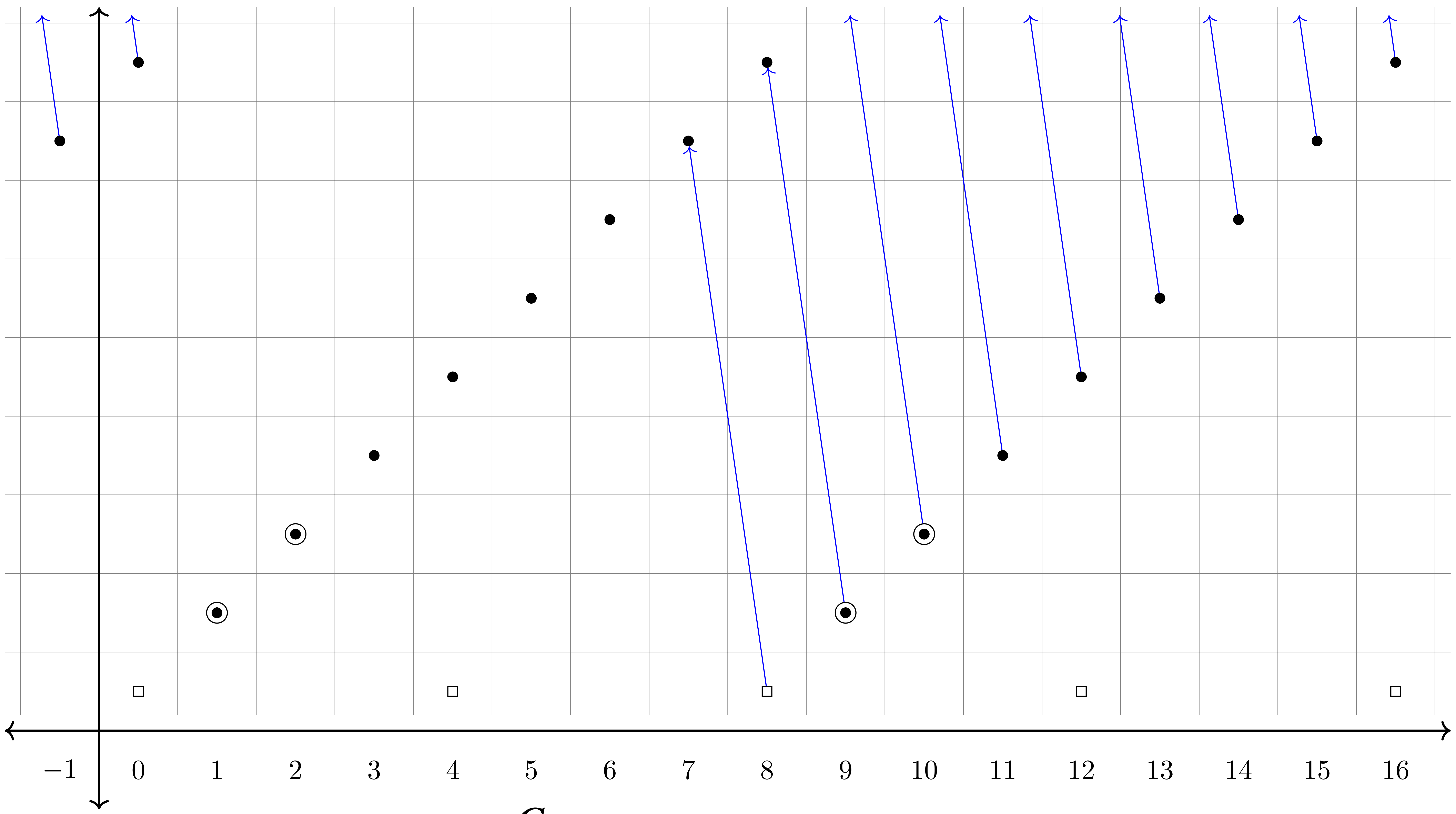
Let $\mathrm{EQ}_{n-1} = (E_n)^{hC_2}$ denote higher hermitian K-theory.

We can detect exotic periodicity!

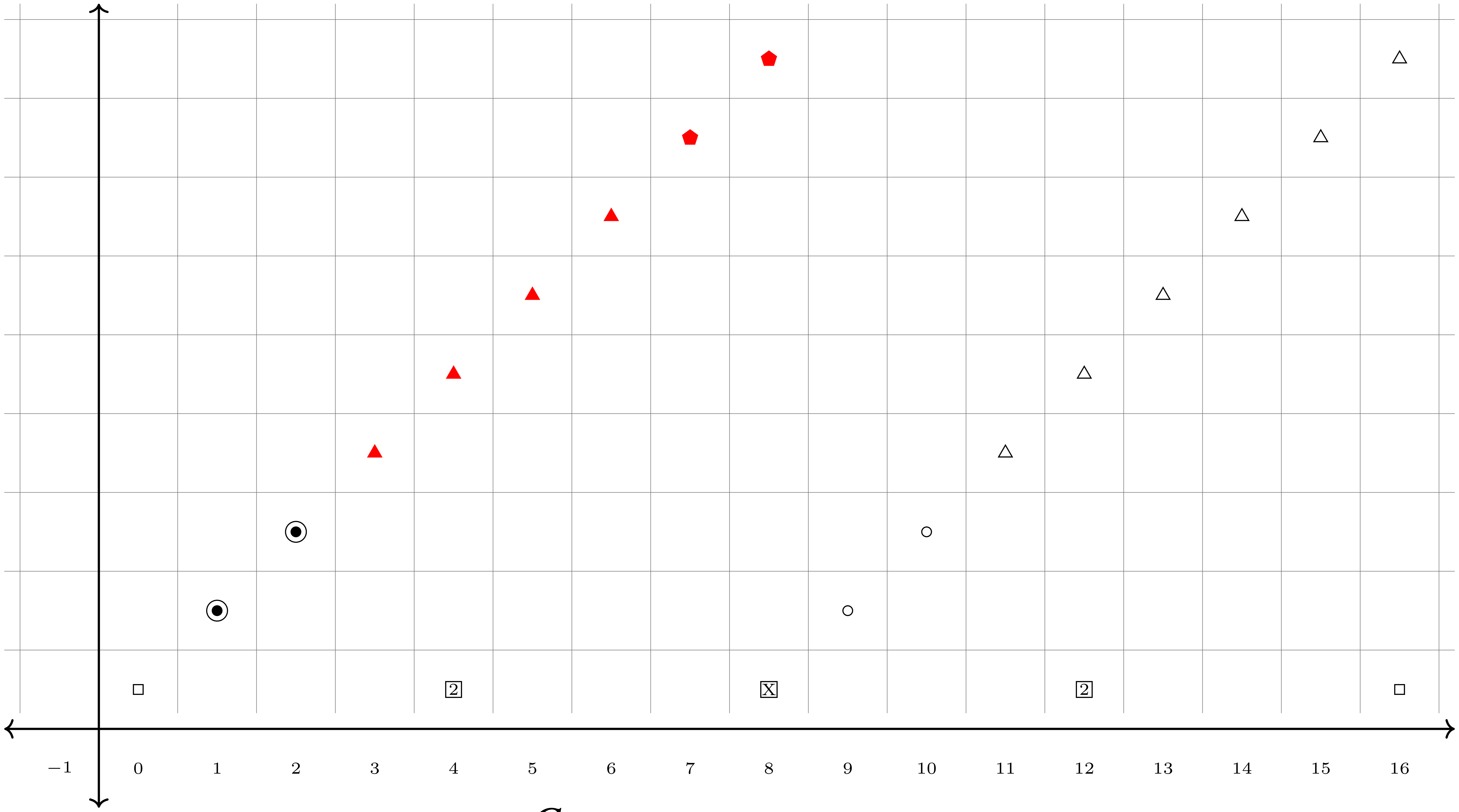
$\mathbb{S} \to \mathrm{EW}_n$



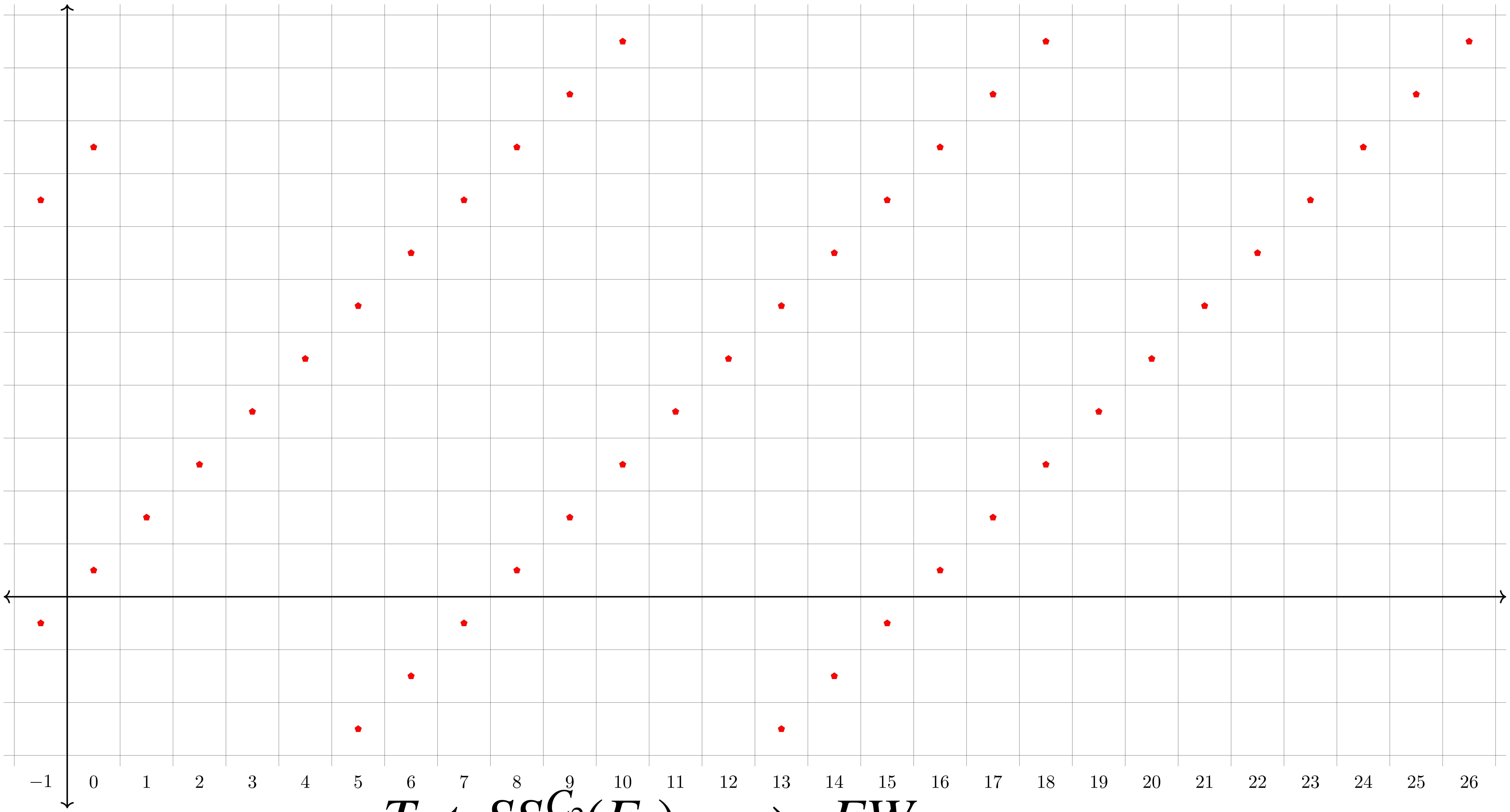
$$HFPSS^{C_2}(E_2) \Longrightarrow EQ_1$$



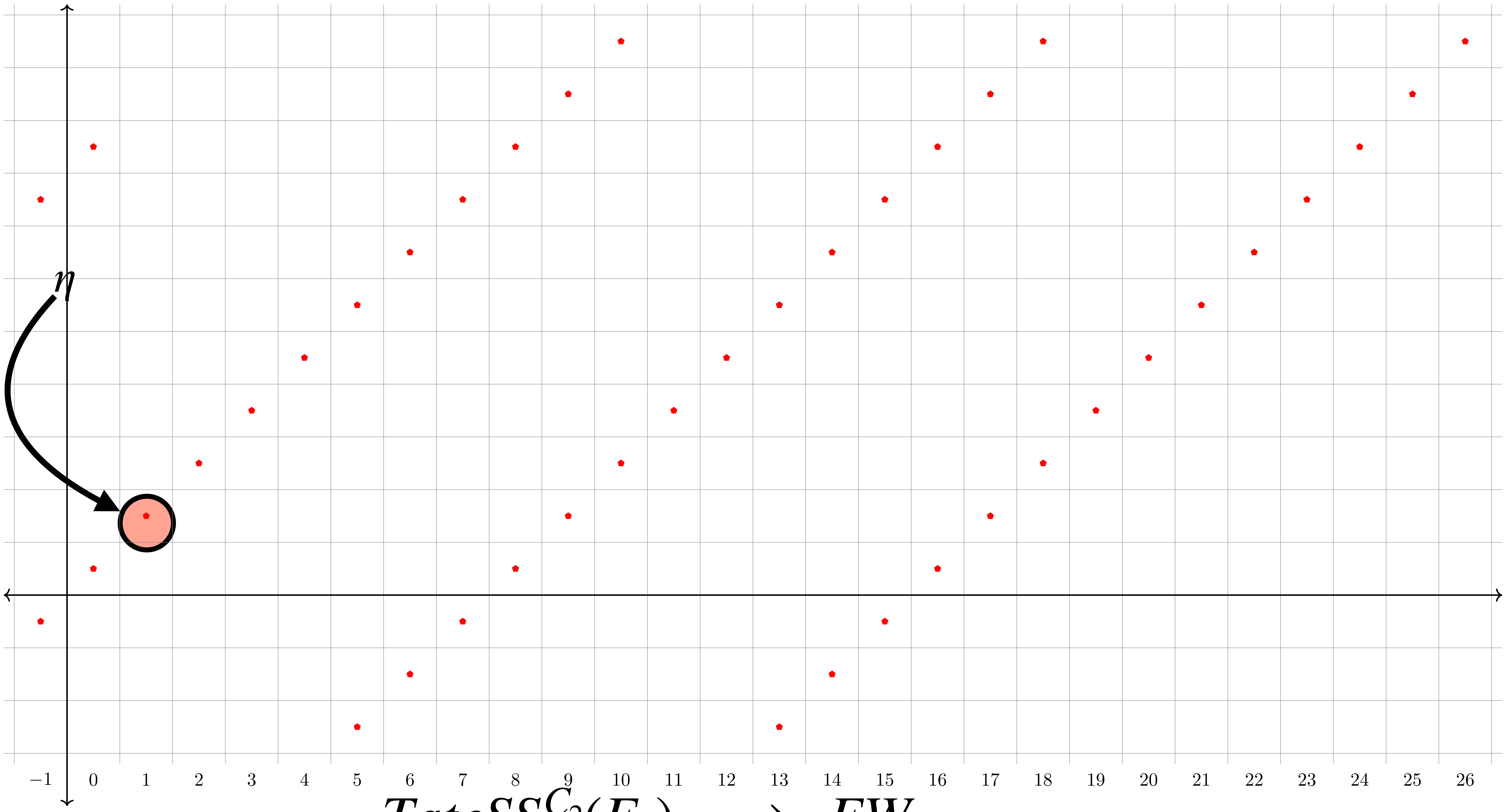
$$HFPSS^{C_2}(E_2) \Longrightarrow EQ_1$$



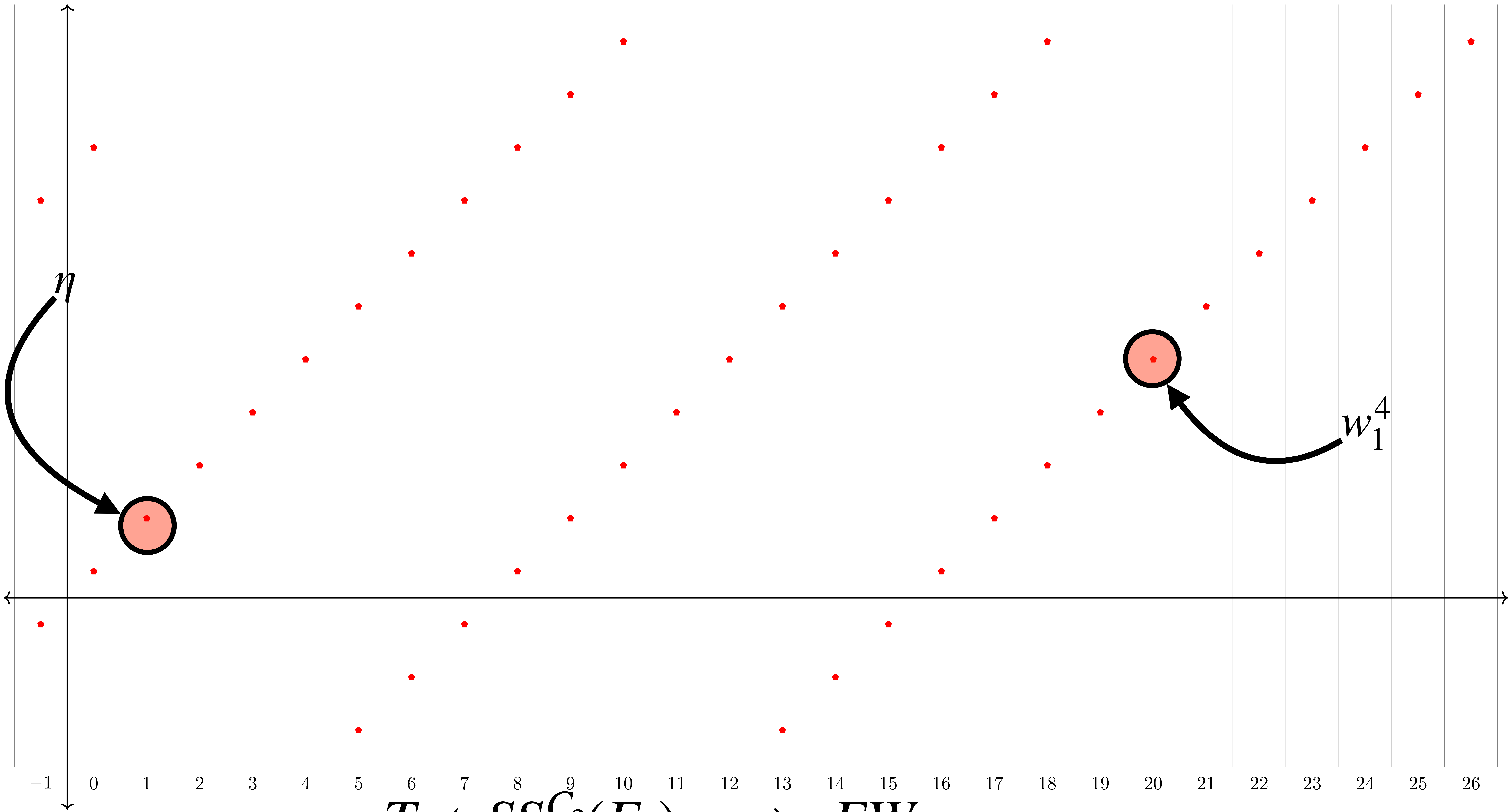
$$HFPSS^{C_2}(E_2) \implies EQ_1$$



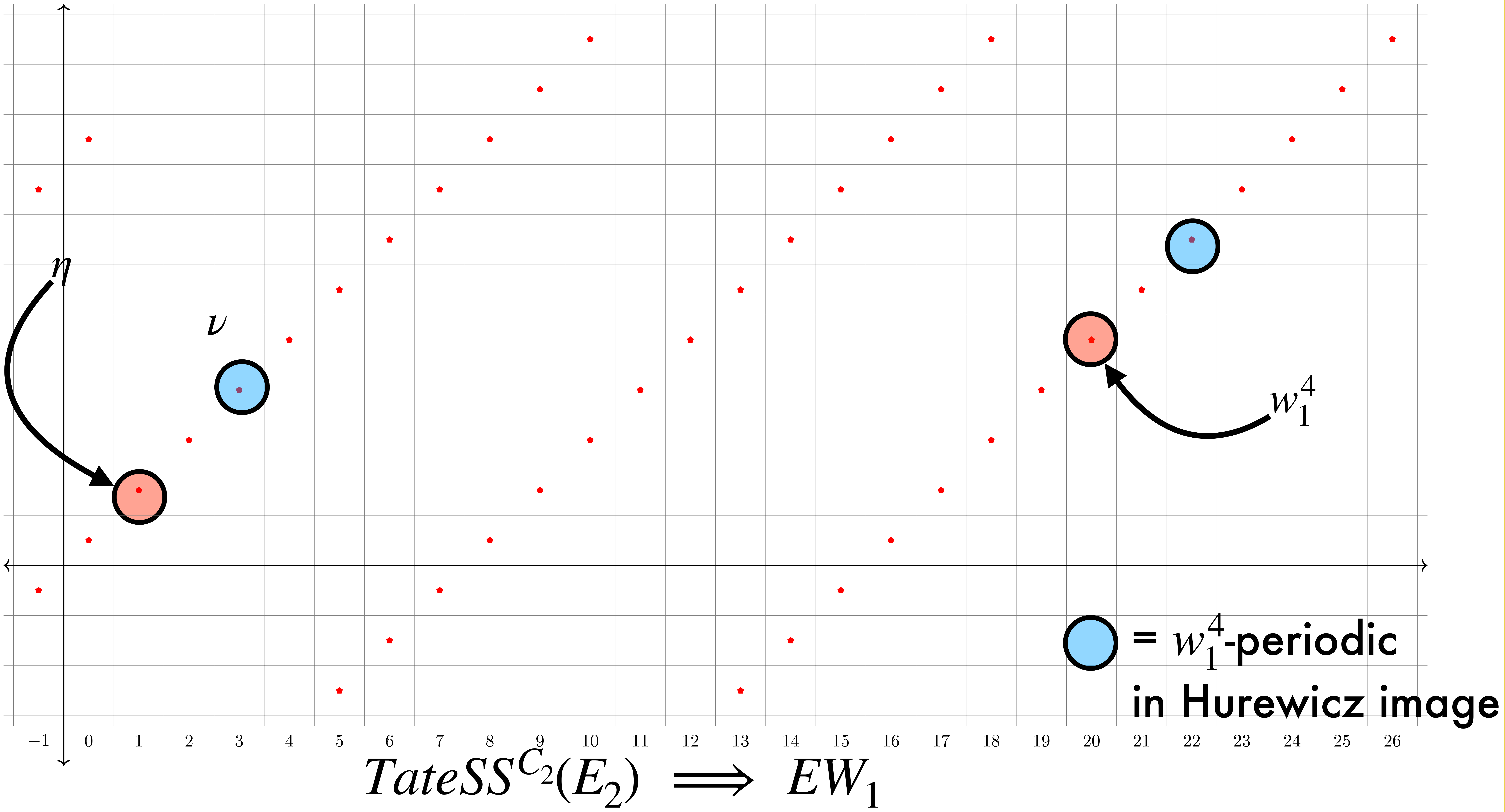
$$TateSS^{C_2}(E_2) \implies EW_1$$



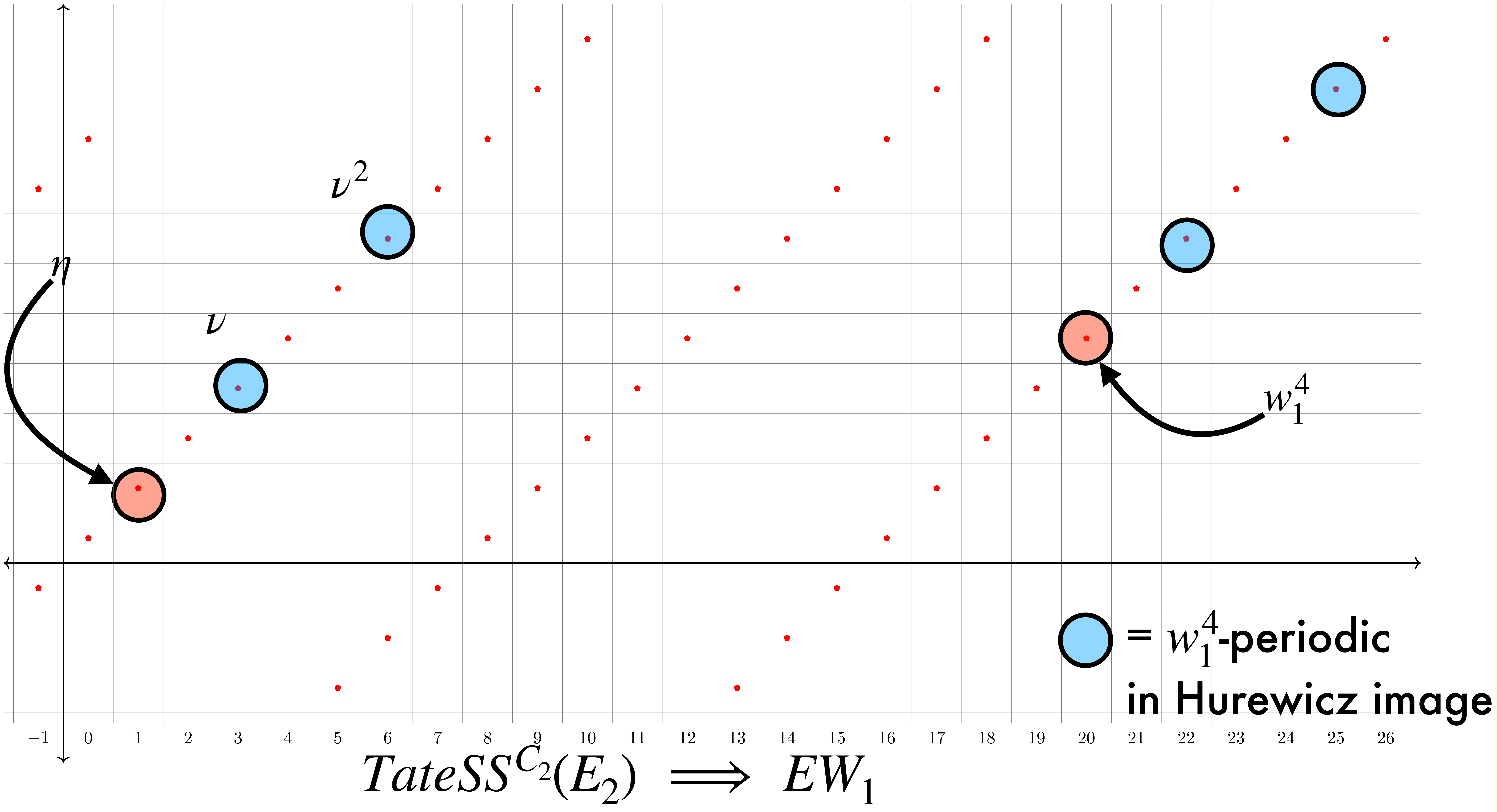
$$TateSS^{C_2}(E_2) \implies EW_1$$



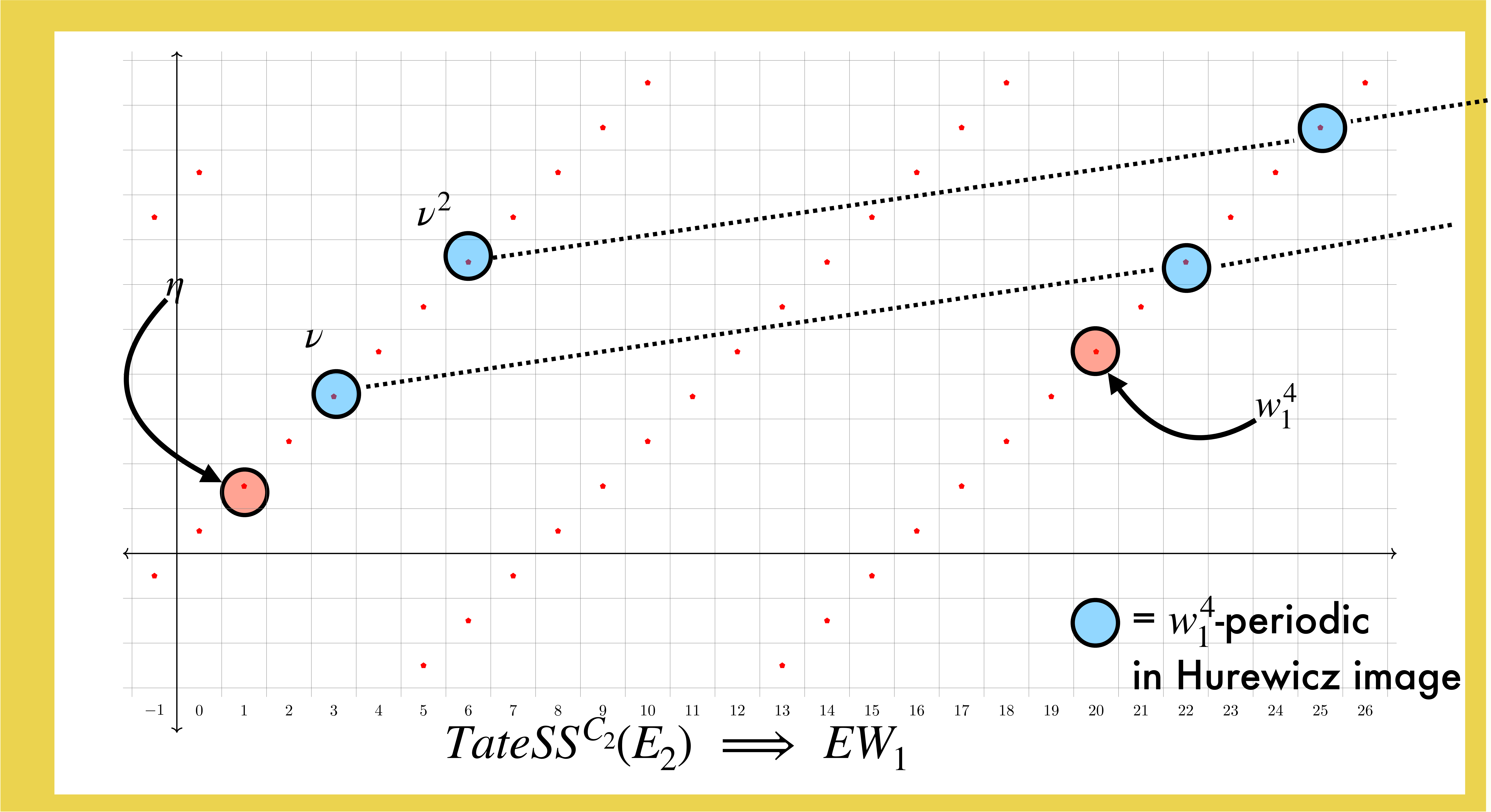
$$TateSS^{C_2}(E_2) \implies EW_1$$



η
ν
w_1^4
= w_1^4-periodic
in Hurewicz image
−1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26
$TateSS^{C_2}(E_2) \implies EW_1$



ν^2
η
ν
w_1^4
= w_1^4-periodic
in Hurewicz image
−1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26
$TateSS^{C_2}(E_2) \implies EW_1$



ν^2
η
ν
w_1^4
= w_1^4-periodic
in Hurewicz image
−1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26
$TateSS^{C_2}(E_2) \implies EW_1$

THANK YOU!